CBSE Class 11 Mathematics

Important Questions

Chapter 12

Introduction to Three Dimensional Geometry

1 Marks Questions

1. Name the octants in which the following lie. (5,2,3)

Ans. I

2. Name the octants in which the following lie. (-5,4,3)

Ans. II

3. Find the image of (-2,3,4) in the y z plane

Ans. (2, 3, 4)

4. Find the image of (5,2,-7) in the xy plane

Ans. (5, 2, 7)

5. A point lie on X –axis what are co ordinate of the point

Ans. $(a, 0, 0)$

6. Write the name of plane in which x axis and y - axis taken together.

Ans. XY Plane

7. The point $(4, -3, -6)$ lie in which octants

Ans. $VIII$

8. The point $(2, 0, 8)$ lie in which plane

Ans. XZ

9. A point is in the XZ plane. What is the value of y co-ordinates?

Ans. Zero

10. What is the coordinates of XY plane

Ans. $(x, y, 0)$

11. The point $(-4, 2, 5)$ lie in which octants.

Ans. II

12. The distance from origin to point (a, b, c) is:

Ans. $\sqrt{a^2 + b^2 + c^2}$

CBSE Class 12 Mathematics
Important Questions
Chapter 12
Introduction to Three Dimensional Geometry

4 Marks Questions

1.Given that P(3,2,-4), Q(5,4,-6) and R(9,8,-10) are collinear. Find the ratio in which Q divides PR

Ans. Suppose Q divides PR in the ratio λ :1. Then coordinator of Q are

$$\left(\frac{9\lambda+3}{\lambda+1}, \frac{8\lambda+2}{\lambda+1}, \frac{-10\lambda-4}{\lambda+1}\right)$$

But, coordinates of Q are (5,4,-6). Therefore

$$\frac{9\lambda+3}{\lambda+1}=5, \frac{8\lambda+2}{\lambda+1}=4, \frac{-10\lambda-4}{\lambda+1}=6$$

These three equations give

$$\lambda=\frac{1}{2}.$$

So Q divides PR in the ratio $\frac{1}{2}:1$ or 1:2

2. Determine the points in xy plane which is equidistant from these point A (2,0,3) B(0,3,2) and C(0,0,1)

Ans. We know that Z- coordinate of every point on xy-plane is zero. So, let $P(x, y, 0)$ be a point in xy-plane such that PA=PB=PC

Now, PA=PB

$\Rightarrow PA^2=PB^2$

$$\Rightarrow (x-2)^2+(y-0)^2+(0-3)^2=(x-0)^2+(y-3)^2+(0-2)^2$$

$$\Rightarrow 4x-6y=0 \text{ or } 2x-3y=0.......(i)$$

$PB=PC$

$$\Rightarrow PB^2=PC^2$$

$$\Rightarrow (x-0)^2+(y-3)^2+(0-2)^2=(x-0)^2+(y-0)^2+(0-1)^2$$

$$\Rightarrow -6y+12=0 \Rightarrow y=2.........(ii)$$

Putting $y=2$ in (i) we obtain $x=3$

Hence the required points (3,2,0).

3. Find the locus of the point which is equidistant from the point A(0,2,3) and B(2,-2,1)

Ans. Let $P(x,y,z)$ be any point which is equidistant from A(0,2,3) and B(2,-2,1). Then

PA=PB

$\Rightarrow PA^2=PB^2$

$$\Rightarrow \sqrt{(x-0)^2+(y-2)^2+(2-3)^2}=\sqrt{(x-2)^2+(y+2)^2+(z-1)^2}$$

$$\Rightarrow 4x-8y-42+4=0 \text{ or } x-2y-2+1=0$$

4. Show that the points A(0,1,2) B(2,-1,3) and C(1,-3,1) are vertices of an isosceles right angled triangle.

Ans. We have

$$AB=\sqrt{(2-0)^2+(-1-1)^2(+3-2)^2}=\sqrt{4+4+1}=3$$

$BC=\sqrt{(1-2)^2+(-3+1)^2+(1-3)^2}=\sqrt{1+4+4}=3$

And $CA=\sqrt{(1-0)^2+(-3-1)^2+(1-2)^2}=\sqrt{1+16+1}=3\sqrt{2}$

Clearly AB=BC and $AB^2+BC^2=AC^2$

Hence, triangle ABC is an isosceles right angled triangle.

5. Using section formula, prove that the three points A(-2,3,5), B(1,2,3), and C(7,0,-1) are collinear.

Ans. Suppose the given points are collinear and C divides AB in the ratio $\lambda:1$.

Then coordinates of C are

$$\left(\frac{\lambda-2}{\lambda+1},\frac{2\lambda+3}{\lambda+1},\frac{3\lambda+5}{\lambda+1}\right)$$

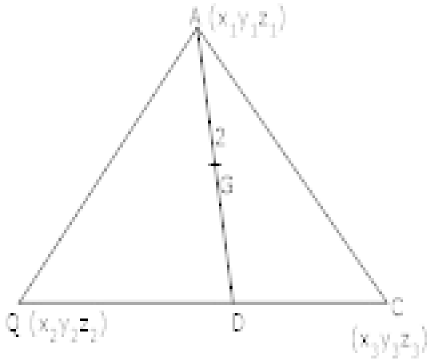
But, coordinates of C are (3,0,-1) from each of there equations, we get $\lambda=\frac{3}{2}$

Since each of there equation give the same value of V. therefore, the given points are collinear and C divides AB externally in the ratio 3:2.

6. Show that coordinator of the centroid of triangle with vertices A($x_1y_1z_1$), B($x_2y_2z_2$), and C($x_3y_3z_3$) is $\left[\frac{x_1+y_1+z_1}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right]$

Ans. Let D be the mid point of AC. Then

Coordinates of D are $\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2},\frac{z_2+z_3}{2}\right)$.



Let G be the centroid of ΔABC. Then G, divides AD in the ratio 2:1. So coordinates of D are

$$\left(\frac{1.x_1 + 2\frac{(x_2+x_3)}{2}}{1+2}, \frac{1.y_1 + 2\left(\frac{y_2+y_3}{2}\right)}{1+2}, \frac{1.z_1 + 2\left(\frac{z_2+z_3}{2}\right)}{1+2} \right)$$

i.e. $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3} \right)$

7. Prove by distance formula that the points $A(1,2,3)$, $B(-1,-1,-1)$ **and** $C(3,5,7)$ **are collinear.**

Ans. Distance

$$|AB| = \sqrt{(-1-1)^2 + (-1-2)^2 + (-1-3)^2} = \sqrt{4+9+16} = \sqrt{29}$$

Distance

$$|BC| = \sqrt{(3+1)^2 + (5+1)^2 + (7+1)^2} = \sqrt{16+36+64} = 2\sqrt{29}$$

Distance

$$|AC| = \sqrt{(3-1)^2 + (5-2)^2 + (7-3)^2} = \sqrt{4+9+16} = \sqrt{29}$$

$\therefore\ |BC| = |AB| + |Ac|$

$\therefore$ The paints A.B.C. are collinear.

8. Find the co ordinate of the point which divides the join of $P(2,-1,4)$ and $Q(4,3,2)$ in the ratio $2:5$ (i) internally (ii) externally

Ans. Let paint $R(x,y,z)$ be the required paint.

(i) For internal division

$$x=\frac{2\times4+5\times2}{2+5}=\frac{8+10}{7}=\frac{18}{7}$$

$$y=\frac{2\times3+5\times-1}{2+5}=\frac{6-5}{7}=\frac{1}{7}$$

$$z=\frac{2\times2+5\times4}{2+5}=\frac{4+20}{7}=\frac{24}{7}$$

$\therefore$ Required paint $R\left(\frac{18}{7},\frac{1}{7},\frac{24}{7}\right)$

(ii) For external division.

$$x=\frac{2\times4-5\times2}{2-5}=\frac{8-10}{-3}=\frac{-2}{-3}=\frac{2}{3}$$

$$y=\frac{2\times3-5\times-1}{2-5}=\frac{6+5}{-3}=\frac{11}{-3}$$

$$z=\frac{2\times2-5\times4}{2-5}=\frac{4-20}{-3}=\frac{-16}{-3}=\frac{16}{3}$$

$\therefore$ Required point $R\left(\frac{2}{3},\frac{-11}{3},\frac{16}{3}\right)$

9. Find the co ordinate of a point equidistant from the four points

$0(0,0,0)$, $A(a,0,0)$, $B(0,b,0)$ **and** $C(0,0,c)$

Ans. Let $P(x, y, z)$ be the required point

According to condition

$OP = PA = PB = PC$

Now $OP = PA$

$\Rightarrow OP^2 = PA^2$

$\Rightarrow x^2 + y^2 + z^2 = (x-a)^2 + (y-0)^2 + (z-0)^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 - 2ax + a^2 + y^2 + z^2$

$2ax = a^2$

$\therefore x = \frac{a}{2}$

Similarly $OP = PB$

$\Rightarrow y = \frac{b}{2}$

$A(x, y, z)$ $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ D, E and F are mid points of side $BC, CA,$ and AB respectively,

Then $\frac{x_1 + x_2}{2} = -1$

$x_1 + x_2 = -2$......(1)

$\frac{y_1 + y_2}{2} = 1$

$y_1 + y_2 = 2 ......(2)$

$\frac{z_1 + z_2}{2} = -4$

$z_1 + z_2 = -8 ......(3)$

$\frac{x_2 + x_3}{2} = 1$

$x_2 + x_3 = 2 ......(4)$

$\frac{y_2 + y_3}{2} = 2$

$y2 + y3 = 4 ......(5)$

$\frac{z_2 + z_3}{2} = -3$

$z_2 + z_3 = -6 ......(6)$

$\frac{x_1 + x_3}{2} = 3$

$x_1 + x_3 = 6 ......(7)$

$\frac{y_1 + y_3}{2} = 0$

$y_1 + y_3 = 0 ......(8)$

$\frac{z_1 + z_3}{2} = 1$

$z_1 + z_3 = 2 ......(9)$

Adding eq (1),(4) and (7) we get

$2(x_1+x_2+x_3)=-2+2+6$

Adding eq. (2),(5) and (8)

$2(y_1+y_2+y_3)=6$

$y_1+y_2+y_3=3......(11)$

And $OP=PC$

$\Rightarrow z=\frac{c}{2}$

Hence co-ordinate of $P\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$

10. Find the ratio in which the join the $A(2,1,5)$ and $B(3,4,3)$ is divided by the plane $2x+2y-2z=1$ Also find the co-ordinate of the point of division

Ans. Suppose plane $2x+2y-2z=1$ divides $A(2,1,5)$ and $B(3,4,5)$ in the ratio $\lambda:1$ at pain C

Then co-ordinate of paint C

$\left(\frac{3\lambda+2}{\lambda+1},\frac{4\lambda+1}{\lambda+1},\frac{3\lambda+5}{\lambda+1}\right)$

$\because$ Point C lies on the plane $2x+2y-2z=1$

$\therefore$ Points C must satisfy the equation of plane

$2\left(\frac{3\lambda+2}{\lambda+1}\right)+2\left(\frac{4\lambda+1}{\lambda+1}\right)-2\left(\frac{3\lambda+5}{\lambda+1}\right)=1$

$\Rightarrow 8\lambda-4=\lambda+1$

$\Rightarrow \lambda = \frac{5}{7}$

$\therefore$ Required ratio 5:7

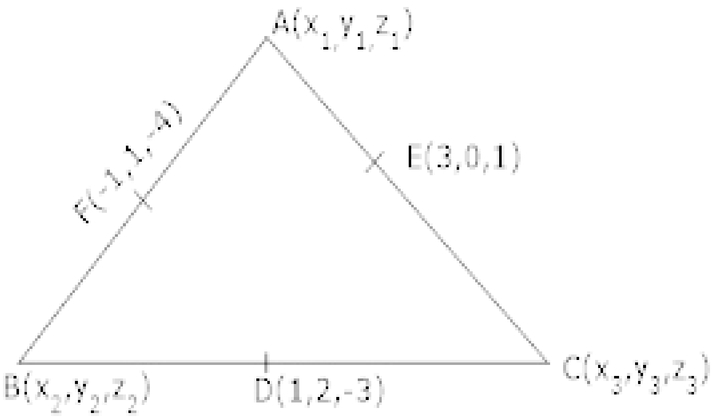
11. Find the centroid of a triangle, mid points of whose sides are $(1,2,-3)$, $(3,0,1)$ and $(-1,1,-4)$

Ans. Suppose co-ordinate of vertices of ΔABC are

Adding eq. (3), (6) and (9)

$$2(z_1+z_2+z_3) = -8-6+2$$

$$z_1+z_2+z_3 = -6......(12)$$



Co-ordinate of centroid

$$x = \frac{x_1+x_2+x_3}{3} = \frac{3}{3} = 1$$

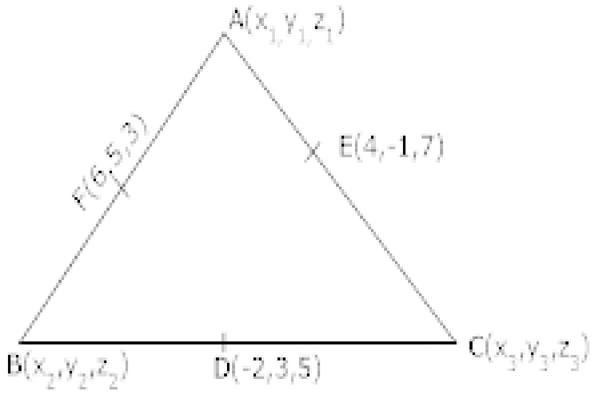
$$y = \frac{y_1+y_2+y_3}{3} = \frac{3}{3} = 1$$

$$z = \frac{z_1+z_2+z_3}{3} = \frac{-6}{3} = -2$$

$(1, 1, -2)$

12. The mid points of the sides of a ΔABC are given by $(-2,3,5)$, $(4,-1,7)$ and $(6,5,3)$ find the co ordinate of A, B and C

Ans. Suppose co-ordinate of point A, B, C, are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) respectively let D, E and F are mid points of side BC, CA and AB respectively



$$\therefore \frac{x_1 + x_2}{2} = 6$$

$$x_1 + x_2 = 12 \ldots\ldots(1)$$

$$\frac{y_1 + y_2}{2} = 5$$

$$y_1 + y_2 = 10 \ldots\ldots(2)$$

$$\frac{z_1 + z_2}{2} = 3$$

$$z_1 + z_2 = 6 \ldots\ldots(3)$$

$$\frac{x_2 + x_3}{2} = -2$$

$$x_2 + x_3 = -4 \ldots\ldots(4)$$

$$\frac{y_2 + y_3}{2} = 3$$

$$y_2 + y_3 = 6 \ldots\ldots(5)$$

$$\frac{z_1 + z_2}{2} = 5$$

$$z_1 + z_2 = 10 \ldots\ldots(6)$$

$$\frac{x_1 + x_3}{2} = 4$$

$$x_1 + x_3 = 8 \ldots\ldots(7)$$

$$\frac{y_1 + y_3}{2} = -1$$

$$y_1 + y_3 = -2 \ldots\ldots(8)$$

$$\frac{z_1 + z_3}{z} = 7$$

$$z_1 + z_3 = 14 \ldots\ldots(9)$$

Adding eq. (1), (4) and (7)

$$2(x_1 + x_2 + x_3) = 12 - 4 + 8$$

$$x_1 + x_2 + x_3 = \frac{16}{2} = 8 \ldots\ldots(10)$$

Similarly $y_1 + y_2 + y_3 = 7 \ldots\ldots(11)$

$$z_1 + z_2 + z_3 = 15 \ldots\ldots(12)$$

Subtracting eq. (1), (4) and (7) from (10)

$$x_3 = -4, \quad x_1 = 12, \quad x_2 = 0$$

Now subtracting eq. (2), (5) and (8) from (11)

$$y_3 = -3, \quad y_1 = 1, \quad y_2 = 9$$

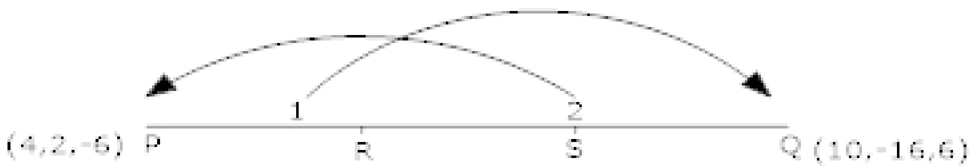
Similarly $z_3 = 9, \quad z_1 = 5, \quad z_2 = 1$

$\therefore$ co-ordinate of point A, B and C are

$A(12, 0, -4)$, $B(1, 9, -3)$, and $C(5, 1, 9)$

13. Find the co-ordinates of the points which trisects the line segment PQ formed by joining the point $P(4, 2, -6)$ and $Q(10, -16, 6)$

Ans. Let R and S be the points of trisection of the segment PO. Then



$\therefore PR = RS = SQ$

$\Rightarrow 2PR = RQ$

$\Rightarrow \frac{PQ}{RQ} = \frac{1}{2}$

$\therefore$ R divides PQ in the ratio 1:2

$\therefore$ Co-ordinates of point

$$R\left[\frac{1(10)+2\times4}{1+2}, \frac{1(-16)+2\times2}{1+2}, \frac{1\times6+2(-6)}{1+2}\right]$$

$= R(6, -4, -2)$

Similarly $PS = 2SQ$

$\Rightarrow \frac{PS}{SQ} = \frac{2}{1}$

$\therefore$ S divider PQ in the ratio 2:1

$\therefore$ co-ordinates of point S

$$\left[\frac{2(10)+1(4)}{1+2}, \frac{2(-16)+1(2)}{1+2}, \frac{2(6)+1(-6)}{1+2}\right]$$

$\therefore S(8,-10,2)$

14. Show that the point $P(1,2,3)$, $Q(-1,-2,-1)$, $R(2,3,2)$ and $S(4,7,6)$ taken in order form the vertices of a parallelogram. Do these form a rectangle?

Ans. Mid point of PR is $\left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right)$

i.e. $\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$

also mid point of QS is $\left(\frac{-1+4}{2}, \frac{-2+7}{2}, \frac{-1+6}{2}\right)$

i.e. $\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$

Then PR and QS have same mid points.

$\therefore$ PR and QS bisect each other. It is a Parallelogram.

Now $PR = \sqrt{(2-1)^2+(3-2)^2+(2-3)^2} = \sqrt{3}$ and

$QS = \sqrt{(4+1)^2+(7+2)^2+(6+1)^2} = \sqrt{155}$

$\therefore PR \neq QS$ diagonals an not equal

$\therefore PQRS$ are not rectangle.

15. A point R with x co-ordinates 4 lies on the line segment joining the points $P(2,-3,4)$ and $Q(8,0,10)$ find the co-ordinates of the point R

Ans. Let the point. R divides the line segment joining the point P and Q in the ratio $\lambda : 1$, Then co-ordinates of Point R

$$\left[\frac{8\lambda+2}{\lambda+1}, \frac{-3}{\lambda+1}, \frac{10\lambda+4}{\lambda+1}\right]$$

The x co-ordinates of point R is 4

$$\Rightarrow \frac{8\lambda+2}{\lambda+1} = 4 \quad , \lambda = \frac{1}{2}$$

$\therefore$ co-ordinates of point R

$$\left[4, \frac{-3}{\frac{1}{2}+1}, \frac{10\times\frac{1}{2}+4}{\frac{1}{2}+1}\right] \qquad \text{i.e.}(4, -2, 6)$$

16. If the points $P(1, 0, -6)$, $Q(-3, P, q)$ and $R(-5, 9, 6)$ are collinear, find the values of P and q

Ans. Given points

$P(1, 0, -6)$, $Q(-3, P, q)$ and $R(-5, 9, 6)$ are collinear

Let point Q divider PR in the ratio K:1

$\therefore$ co-ordinates of point $P\left(\frac{1-5K}{K+1}, \frac{0+9K}{K+1}, \frac{-6+6K}{K+1}\right)$

$Q(-3, P, q)$

$$\frac{1-5K}{K+1}=-3$$

$$1-5K=-3K-3$$

$$-2K=-4$$

$$K=\frac{-4}{-2}$$

$$K=2$$

$\therefore$ the value of P and q are 6 and 2.

17. Three consecutive vertices of a parallelogram ABCD are $A(3,-1,2)$, $B(1,2,-4)$ and $C(-1,1,2)$ find forth vertex D

Ans. Given vertices of 11gm ABCD

$A(3,-1,2)$, $B(1,2,-4)$, $C(-1,1,2)$

Suppose co-or dine of forth vertex $D(x,y,z)$

Mid point of $AC\left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)$

$=(1,0,2)$

Mid point of $BD\left(\frac{x+1}{2},\frac{y+2}{2},\frac{-4+z}{2}\right)$

Mid point of AC = mid point of BD

$$\frac{x+1}{2}=1\Rightarrow x=1$$

$$\frac{y+2}{2}=0\Rightarrow y=-2$$

$$\frac{-4+z}{2}=2 \Rightarrow z=8$$

Co-ordinates of point $D(1,-2,8)$

18. If A and B be the points $(3,4,5)$ and $(-1,3,7)$ respectively. Find the eq. of the set points P such that $PA^2+PB^2=K^2$ where K is a constant

Ans. Let co-ordinates of point P be

(x,y,z)

$PA^2=(x-3)^2+(y-4)^2+(z-5)^2$

$=x^2-6x+9+y^2-8y+16+z^2-10z+25$

$=x^2+y^2+z^2-6x-8y-10z+50$

$PB^2=(x+1)^2+(y-3)^2+(z-7)^2$

$=x^2+2x+1+y^2-6y+9+z^2-14+49$

$=x^2+y^2+z^2+2x-6y-14z+59$

$PA^2+PB^2=K^2$

$2(x^2+y^2+z^2)-4x-14y-24z+109=K^2$

$$x^2+y^2+z^2-2x-7y-12z=\frac{K^2-109}{2}$$

CBSE Class 12 Mathematics

Important Questions

Chapter 12

Introduction to Three Dimensional Geometry

6 Marks Questions

1. Prove that the lines joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent.

Ans. Let ABCD be tetrahedron such that the coordinates of its vertices are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$

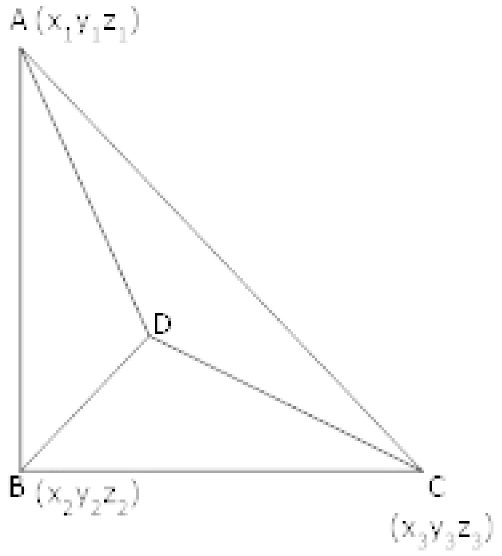
The coordinates of the centroids of faces ABC, DAB, DBC and DCA respectively

$$G_1\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right]$$

$$G_2\left[\frac{x_1+x_2+x_4}{3}, \frac{y_1+y_2+y_4}{3}, \frac{z_1+z_2+z_4}{3}\right]$$

$$G_3\left[\frac{x_2+x_3+x_4}{3}, \frac{y_2+y_3+y_4}{3}, \frac{z_2+z_3+z_4}{3}\right]$$

$$G_4\left[\frac{x_4+x_3+x_1}{3}, \frac{y_4+y_3+y_1}{3}, \frac{z_4+z_3+z_1}{3}\right]$$



Now, coordinates of point G dividing DG1 in the ratio 3:1 are

$$\left[\frac{1.x_4+3\left(\frac{x_1+x_2+x_3}{3}\right)}{1+3},\frac{1.y_4+3\left(\frac{y_1+y_2+y_3}{3}\right)}{1+3},\frac{1.z_4+3\left(\frac{z_1+z_2+z_3}{3}\right)}{1+3}\right]$$

$$=\left[\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right]$$

Similarly the point dividing CG2, AG3 and BG4 in the ratio 3:1 has the same coordinates.

Hence the point $G\left[\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right]$ is common to DG1, CG2, AG3 and BG4.

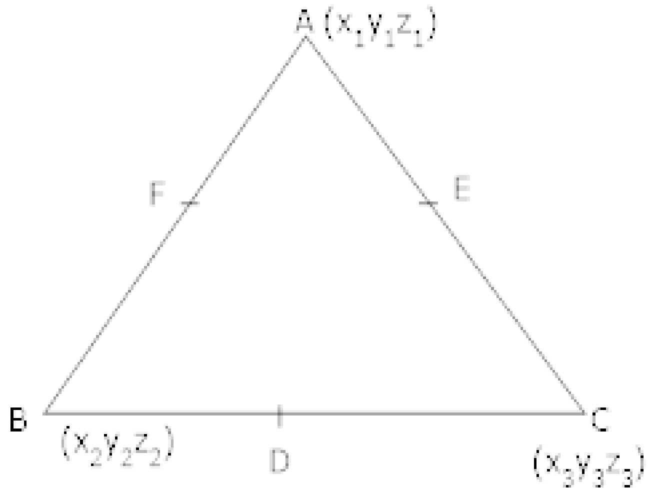
Hence they are concurrent.

2. The mid points of the sides of a triangle are (1,5,-1), (0,4,-2) and (2,3,4). Find its vertices.

Ans. Suppose vertices of ΔABC are $A(x_1y_1z_1)$, $B(x_2y_2z_2)$ and $C(x_3y_3z_3)$ respectively

Given coordinates of mid point of side BC, CA, and AB respectively are D(1,5,-1), E(0,4,-2) and

F(2,3,4)



$\therefore \frac{x_2+x_3}{2}=1 \quad \frac{y_2+y_3}{2}=5 \quad \frac{z_2+z_3}{2}=-1$

$x_2+x_3=2......(i)$

$\frac{x_1+x_3}{2}=0$

$y_2+y_3=10......(ii)$

$\frac{y_1+y_3}{2}=4$

$z_2+z_3=2......(iii)$

$\frac{z_1+z_3}{2}=-2$

$x_1+x_3=0.......(iv)$

$\frac{x_1+x_2}{2}=2$

$y_1+y_2=8.......(v)$

$\frac{y_1+y_2}{2}=3$

$z_1 + z_3 = -4 \ldots\ldots.(vi)$

$\frac{z_1 + z_2}{2} = 4$

$x_1 + x_2 = 4 \ldots\ldots.(vii)$

$y_1 + y_2 = 6 \ldots\ldots.(viii)$

$z_1 + z_2 = 8 \ldots\ldots.(ix)$

Adding eq. $(i),(iv),\&(vii)$

$2(x_1 + x_2 + x_3) = 6$

$x_1 + x_2 + x_3 = 3 \ldots\ldots.(x)$

Subtracting eq. $(i),(iv),\&(vii)$ from (x) we get

$x_1 = 1,\ \ x_2 = 3,\ \ x_3 = -1$

Similarly, adding eq. $(ii),(v)$ and $(viii)$

$y_1 + y_2 + y_3 = 12 \ldots\ldots(xi)$

Subtracting eq. $(ii),(v)$ and $(viii)$ from (xi)

$y_1 = 2,\ \ y_2 = 4,\ \ y_3 = 6$

Similarly $z_1 + z_2 + z_3 = 3$

$z_1 = 1,\ \ z_2 = 7,\ \ z_3 = -5$

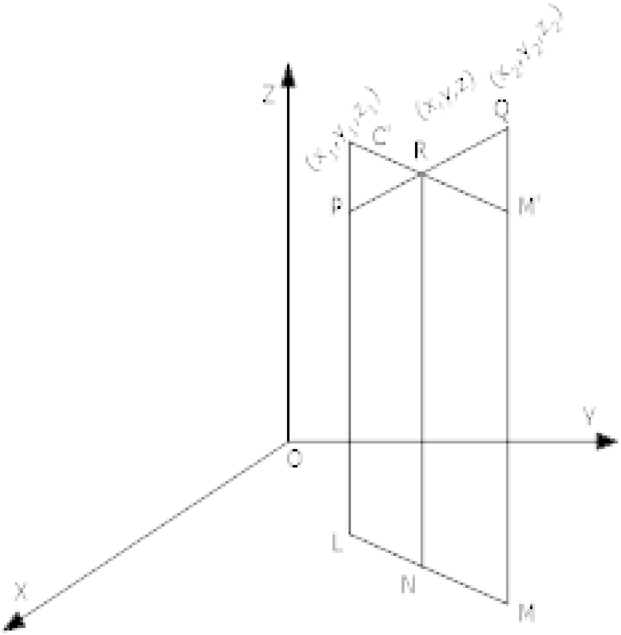
$\therefore$ Coordinates of vertices of Δ ABC are A(1,3,-1), B(2,4,6) and C(1,7,-5)

3. Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points in space find co ordinate of point R

which divides P and Q in the ratio $m_1 : m_2$ by geometrically

Ans. Let co-ordinate of Point R be (x, y, z) which divider line segment joining the point $P\ Q$ in the ratio $m_1 : m_2$

Clearly $\Delta PRL' \sim \Delta QRM'$ $[By\ AA \text{ sinilsrity}]$



$$\therefore \frac{PL'}{MQ'} = \frac{PR}{RQ}$$

$$\Rightarrow \frac{LL' - LP}{MQ - MM'} = \frac{m_1}{m_2}$$

$$\Rightarrow \frac{NR - LP}{MQ - NR} = \frac{m_1}{m_2} \qquad \begin{bmatrix} \because LL' = NR \\ \text{and } MM' = NR \end{bmatrix}$$

$$\Rightarrow \frac{z - z_1}{z_2 - z} = \frac{m_1}{m_2}$$

$$\Rightarrow z = \frac{m_1 z_2 + m_2 z_1}{m_1 + m_2}$$

Similarly $x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$ and

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

4. Show that the plane $ax + by + cz + d = 0$ divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}$ s

Ans. Suppose the plane $ax + by + cz + d = 0$ divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $\lambda : 1$

$$\therefore\ x = \frac{\lambda x_2 + x_1}{\lambda + 1}, \quad y = \frac{\lambda y_2 + y_1}{\lambda + 1}, \quad z = \frac{\lambda z_2 + z_1}{\lambda + 1}$$

$\because$ Plane $ax + by + cz + d = 0$ Passing through (x, y, z)

$$\therefore\ Q\frac{(\lambda x_2 + x_1)}{\lambda + 1} + b\frac{(\lambda y_2 + y_1)}{\lambda + 1} + c\frac{(\lambda z_2 + z_1)}{\lambda + 1} + d = 0$$

$$a(\lambda x_2 + x_1) + b(\lambda y_2 + y_1) + c(\lambda z_2 + z_1) + d(\lambda + 1) = 0$$

$$\lambda(ax_2 + by_2 + cz_2 + d) + (ax_1 + by_1 + cz_1 + d) = 0$$

$$\lambda = -\frac{(ax_1 + by_1 + cz_1 + d)}{(ax_2 + by_2 + cz_2 + d)}$$

Hence Proved.

5. Prove that the points $0(0, 0, 0)$, $A(2, 0, 0)$, $B(1, \sqrt{3}, 0)$, and $C\left(1, \frac{1}{\sqrt{3}}, \frac{2\sqrt{2}}{\sqrt{3}}\right)$ are the vertices of a regular tetrahedron.

Ans. To prove O, A, B, C are vertices of regular tetrahedron.

We have to show that

$|OA| = |OB| = |OC| = |AB| = |BC| = |CA|$

$|OA| = \sqrt{(0-2)^2 + 0^2 + 0^2} = 2$ unit

$|OB| = \sqrt{(0-1)^2 + \left(0-\sqrt{3}\right)^2 + 0^2} = \sqrt{1+3} = \sqrt{4} = 2$ unit

$$|OC| = \sqrt{(0-1)^2 + \left(0 - \frac{1}{\sqrt{3}}\right) + \left(0 - \frac{2\sqrt{2}}{3}\right)^2}$$

$$= \sqrt{1 + \frac{1}{3} + \frac{8}{3}}$$

$$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2 \text{ unit}$$

$|AB| = \sqrt{(2-1)^2 + \left(0-\sqrt{3}\right)^2 + (10-0)^2} = \sqrt{1+3+0}$

$= \sqrt{4} = 2$ unit

$$|BC| = \sqrt{(1-1)^2 + \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2 + \left(0 - \frac{2\sqrt{2}}{\sqrt{3}}\right)^2}$$

$$= \sqrt{0 + \left(\frac{2}{\sqrt{3}}\right)^2 + \frac{8}{3}}$$

$$= \sqrt{\frac{12}{3}} = 2 \text{ unit}$$

$|CA| = \sqrt{(1-2)^2 + \left(\frac{1}{\sqrt{3}} - 0\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{3}} - 0\right)^2}$

$= \sqrt{1 + \frac{1}{3} + \frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = 2$ unit

∴ |AB| = |BC| = |CA| = |OA| = |OB| = |OC| = 2 unit

∴ O, A, B, C are vertices of a regular tetrahedron.

6. If A and B are the points $(-2, 2, 3)$ and $(-1, 4, -3)$ respectively, then find the locus of P such that 3|PA| = 2|PB|

Ans. Given points $A(-2, 2, 3)$ and $B(-1, 4, -3)$

Supper co-ordinates of point $P(x, y, z)$

$|PA| = \sqrt{(x+2)^2 + (y-2)^2 + (2-3)^2}$

$|PA| = \sqrt{x^2 + y^2 + z^2 + 4x - 4y - 6z + 17}$

$|PB| = \sqrt{(x+1)^2 + (y-4)^2 + (z+3)^2}$

$|PB| = \sqrt{x2 + y2 + z2 + 2x - 8y + 6z + 26}$

∵ 3|PA| = 2|PB|

9 PA2=4 PB2

$9(x^2 + y^2 + z^2 + 4x - 4y - 6z + 17) = 4(x^2 + y^2 + z^2 + 2x - 8y + 6z + 26)$

$5x^2 + 5y^2 + 5z^2 + 28x - 4y - 30z + 49 = 0$